NAG Toolbox for MATLAB

f08ah

1 Purpose

f08ah computes the LQ factorization of a real m by n matrix.

2 Syntax

```
[a, tau, info] = f08ah(a, 'm', m, 'n', n)
```

3 Description

f08ah forms the LQ factorization of an arbitrary rectangular real m by n matrix. No pivoting is performed.

If $m \leq n$, the factorization is given by:

$$A = (L \quad 0)Q$$

where L is an m by m lower triangular matrix and Q is an n by n orthogonal matrix. It is sometimes more convenient to write the factorization as

$$A = (L \quad 0)\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$$

which reduces to

$$A = LQ_1,$$

where Q_1 consists of the first m rows of Q, and Q_2 the remaining $n - m$ rows.

If $m > n$, L is trapezoidal, and the factorization can be written

$$A = \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} Q$$

where L_1 is lower triangular and L_2 is rectangular.

The LQ factorization of A is essentially the same as the QR factorization of A^{T}, since

$$A = (L \quad 0)Q \Leftrightarrow A^{\mathrm{T}} = Q^{\mathrm{T}}\begin{pmatrix} L^{\mathrm{T}} \\ 0 \end{pmatrix}.$$

The matrix Q is not formed explicitly but is represented as a product of $\min(m, n)$ elementary reflectors (see the F08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 8).

Note also that for any $k < m$, the information returned in the first k rows of the array **a** represents an LQ factorization of the first k rows of the original matrix A.

4 References

None.

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda**,∗) – **double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,$*$) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $m \leq n$, the elements above the diagonal are overwritten by details of the orthogonal matrix Q and the lower triangle contains the corresponding elements of the m by m lower triangular matrix L.

If $m > n$, the strictly upper triangular part contains details of the orthogonal matrix Q and the remaining elements are overwritten by the corresponding elements of the m by n lower trapezoidal matrix L.

2: **tau($*$) – double array**

Note: the dimension of the array **tau** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

further details of the orthogonal matrix Q.

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **a**, 4: **lda**, 5: **tau**, 6: **work**, 7: **lwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed factorization is the exact factorization of a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}m^2(3n - m)$ if $m \leq n$ or $\frac{2}{3}n^2(3m - n)$ if $m > n$.

To form the orthogonal matrix Q f08ah may be followed by a call to f08aj:

```
[a, info] = f08aj(a, tau, 'k', min(m,n));
```

but note that the first dimension of the array **a**, specified by the parameter **lda**, must be at least **n**, which may be larger than was required by f08ah.

When $m \leq n$, it is often only the first m rows of Q that are required, and they may be formed by the call:

```
[a, info] = f08aj(a, tau, 'k', m);
```

To apply Q to an arbitrary real rectangular matrix C, f08ah may be followed by a call to f08ak. For example,

```
[c, info] = f08ak('Left', 'Transpose', a, tau, c, 'k', min(m, n));
```

forms the matrix product $C = Q^{\mathrm{T}}C$, where C is m by p.

The complex analogue of this function is f08av.

9 Example

```
a = [-5.42, 3.28, -3.68, 0.27, 2.06, 0.46;
     -1.65, -3.4, -3.2, -1.03, -4.06, -0.01;
     -0.37, 2.35, 1.9, 4.31, -1.76, 1.13;
     -3.15, -0.11, 1.99, -2.7, 0.26, 4.5];
[aOut, tau, info] = f08ah(a)
```

```
aOut =
    7.6292   -0.2514    0.2820   -0.0207   -0.1579   -0.0353
    0.1207    6.4848    0.2614    0.1033    0.4201    0.0070
    0.1021   -1.6619   -5.4266    0.6051   -0.5387    0.1686
    1.4766    0.1088    0.4223    6.2555   -0.1704   -0.3499
tau =
    1.7104
    1.5929
    1.1871
    1.7369
info =
           0
```